

Global Structure of Black Holes in String Theory with Gauss-Bonnet Correction in Various Dimensions

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Abstract

We study global structures of black hole solutions in Einstein gravity with Gauss-Bonnet term coupled to dilaton in various dimensions. In particular we focus on the problem whether the singularity is weakened due to the Gauss-Bonnet term and dilaton. We find that there appears the non-central singularity between horizon and the center in many cases, where the metric does not diverge but the Kretschmann invariant does diverge. Hence this is a singularity, but we find the singularity is much milder than the Schwarzschild solution and the non-dilatonic one. We discuss the origin of this “fat” singularity. In other cases, we encounter singularity at the center which is much stronger than the usual one. We find that our black hole solutions have three different types of the global structures; the Schwarzschild, Schwarzschild-AdS and “regular AdS black hole” types.

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§1. Introduction

One of the long-standing problems in theoretical physics is how to reconcile gravity with quantum theory. It is well known that Einstein gravity is not renormalizable and there is an intrinsically difficult problem how to understand physical properties near the singularity which always exists in black hole spacetimes given as solutions in Einstein gravity. Superstring theory is the leading candidate for quantum gravity. It is long expected that the theory could resolve this problem. In order to study the geometrical properties and strong gravitational phenomena, it is still difficult to apply full superstring theory itself. In this situation, it is appropriate to investigate these problems by using the effective low-energy field theories including string quantum corrections.

Many works have been done on black hole solutions in dilatonic gravity, and various properties have been studied since the work in Refs. 1) and 2). On the other hand, it is known that there are higher-order quantum corrections from string theories.³⁾ It is thus important to ask how these corrections may modify the results. Several works have studied the effects of higher order terms,^{4)–9)} but most of the work considers theories without dilaton,^{10)–14)} which is one of the most important ingredients in the string effective theories. Hence it is most significant to study black hole solutions and their properties in the theory with the higher order corrections and dilaton. The simplest higher order correction is the Gauss-Bonnet (GB) term coupled to dilaton in heterotic string theories.

In our previous paper,¹⁵⁾ we have studied asymptotically flat black hole solutions with the GB correction term and dilaton without a cosmological constant in various dimensions from 4 to 10 with $(D - 2)$ -dimensional hypersurface of curvature signature $k = +1$. We have then presented our results on black hole solutions with the cosmological constant with $(D - 2)$ -dimensional hypersurface with $k = 0, \pm 1$.^{16)–18)} In the string perspective, it is also interesting to examine asymptotically non-flat black hole solutions with possible application to AdS/CFT and dS/CFT correspondences in mind.^{19)–21)} Discussions of the origin of such cosmological constant are given in Refs. 22) and 23). Extremal solutions in similar systems have been discussed recently in Ref. 24), and other black hole solutions are presented in Ref. 25).

Most of the above works consider the properties of the external spacetime of the black hole horizon except for Refs. 5), 6), 13) and 14). It is expected that the higher order corrections and dilaton significantly affects the internal structure of spacetime and the structures of singularity. For example, for the so-called small black holes, which have the singularity at the horizon without enough number of charges to support the horizon, it is known that the horizon is stretched due to the presence of higher order terms, and the singularity is resolved there.^{26)–29)}

The cosmological constant, on the other hand, is not expected to modify significantly the short distance properties of the black holes, not affecting the singularities much. In this paper, we study this problem for the solutions we already obtained in our series of papers.^{15)–18)} The black hole solutions exist for the cases $(A, k) = (0, 1), (1, 1), (-1, 0), (-1, 1), (-1, 1)$, where A is normalized to 0 and ± 1 . Since the external structures of these black holes are studied already in these papers, we focus on the internal properties in this paper. We have found that cosmological horizons are not formed in the presence of the positive cosmological constant.¹⁸⁾ Hence for the solutions with the cosmological constant, we mainly discuss asymptotically anti-de Sitter (AdS) solutions with the negative cosmological constant.

The most interesting problem is whether the higher order corrections and dilaton make the strength of the singularity weaker. A special feature which occurs in the presence of the GB term without dilaton is that the singularity arises in the intermediate position inside the horizon before reaching the center $r = 0$ for the black holes with negative mass.¹³⁾ The singularity appears slightly weaker than the Schwarzschild solutions. In other cases, the singularity appears at the origin, and the singularity is weaker for $D \geq 5$ (the GB term does not give any effect in $D = 4$). In our dilatonic case, we also find the similar kind of singularity for lower-dimensional cases ($D = 4, 5$) in asymptotically flat solutions and in $k = +1$ and 0 asymptotically AdS solutions. We call this “fat singularity” because it is made fat due to the presence of the GB term and dilaton. However, the formation mechanism of the fat singularity is different from the non-dilatonic one. It is deeply related to the singularity associated with dilaton. In $D = 5$ dimension, the fat singularity occurs for large black holes. The metric itself does not diverge there while Kretschmann invariant diverges. Hence this is a singularity and the spacetime ends there. The singularity itself is not strong compared with the usual central singularity of Schwarzschild solutions and the non-dilatonic case. In all these cases, the singularities are spacelike. In other cases, we encounter the singularity at the origin $r = 0$, just like Schwarzschild solutions. The nature of the central singularity deviates significantly from the non-dilatonic case and the strength of the central singularity is stronger than Schwarzschild solution. We should note that these results are obtained for a particular value of the dilaton coupling $\gamma = 1/2$. Actually we confirm that the occurrence of the fat singularity seems to depend on the dilaton coupling in the GB term. If we adopt the other dilaton coupling $\gamma = 1$, we find that the fat singularity appears in $D = 6$. Note also that the fat singularity always appears in four dimensions. Thus the presence of the dilaton significantly affects the singularity.

This paper is organized as follows. In § 2, we summarize the action and field equations of the theory we discuss. In § 3, we summarize the internal spacetime and global structure of the black holes for the known cases. First in § 3.1, we summarize the singularity in Schwarzschild

solution in general relativity (GR). Then we discuss the non-dilatonic case with the GB term in § 3.2. We go on to study the internal spacetime and the global structure of the dilatonic solutions in § 4. We give the detailed account of these for asymptotically flat solutions in § 4.1, for $\Lambda < 0$ and $k = 0$ case in § 4.2, for $\Lambda < 0$ and $k = 1$ in § 4.3, and finally for $\Lambda < 0$ and $k = -1$ case in § 4.4. In particular, we discuss the origin of the fat singularity. Our conclusions and discussions are given in § 5.

§2. Dilatonic Einstein-Gauss-Bonnet theory

We consider the following low-energy effective action for a heterotic string

$$S = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{-g} \left[R - \frac{1}{2}(\partial_\mu \phi)^2 + \alpha_2 e^{-\gamma\phi} R_{\text{GB}}^2 - \Lambda e^{\lambda\phi} \right], \quad (2.1)$$

where κ_D^2 is a D -dimensional gravitational constant, ϕ is a dilaton field, $\alpha_2 = \alpha'/8$ is a numerical coefficient given in terms of the Regge slope parameter α' , and $R_{\text{GB}}^2 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ is the GB correction. In this paper we take the coupling constant of dilaton $\gamma = 1/2$, the value that the ten-dimensional critical string theory predicts.

We parametrize the metric as

$$ds_D^2 = -B e^{-2\delta} dt^2 + B^{-1} dr^2 + r^2 h_{ij} dx^i dx^j, \quad (2.2)$$

where $h_{ij} dx^i dx^j$ represents the line element of a $(D-2)$ -dimensional hypersurface with constant curvature $(D-2)(D-3)k$ and volume Σ_k for $k = \pm 1, 0$.

The metric function $B = B(r)$ and the lapse function $\delta = \delta(r)$ depend only on the coordinate r . The field equations are^{16), 17)}

$$\begin{aligned} & [(k-B)\tilde{r}^{D-3}]' \frac{D-2}{\tilde{r}^{D-4}} h - \frac{1}{2} B \tilde{r}^2 \phi'^2 - (D-1)_4 e^{-\gamma\phi} \frac{(k-B)^2}{\tilde{r}^2} \\ & + 4(D-2)_3 \gamma e^{-\gamma\phi} B(k-B)(\phi'' - \gamma\phi'^2) \\ & + 2(D-2)_3 \gamma e^{-\gamma\phi} \phi' \frac{(k-B)[(D-3)k - (D-1)B]}{\tilde{r}} - \tilde{r}^2 \tilde{\Lambda} e^{\lambda\phi} = 0, \end{aligned} \quad (2.3)$$

$$\delta'(D-2)\tilde{r}h + \frac{1}{2}\tilde{r}^2\phi'^2 - 2(D-2)_3 \gamma e^{-\gamma\phi} (k-B)(\phi'' - \gamma\phi'^2) = 0, \quad (2.4)$$

$$\begin{aligned} (e^{-\delta}\tilde{r}^{D-2}B\phi')' &= \gamma(D-2)_3 e^{-\gamma\phi-\delta}\tilde{r}^{D-4} \left[(D-4)_5 \frac{(k-B)^2}{\tilde{r}^2} + 2(B' - 2\delta'B)B' \right. \\ & \left. - 4(k-B)BU(r) - 4\frac{D-4}{\tilde{r}}(B' - \delta'B)(k-B) \right] + e^{-\delta}\tilde{r}^{D-2}\lambda\tilde{\Lambda}e^{\lambda\phi}, \end{aligned} \quad (2.5)$$

where we have defined the dimensionless variables: $\tilde{r} \equiv r/\sqrt{\alpha_2}$, $\tilde{\Lambda} = \alpha_2 \Lambda$, and the primes in the field equations denote the derivatives with respect to \tilde{r} . Namely we measure our length in the unit of $\sqrt{\alpha_2}$. We will introduce the AdS radius ℓ and the mass of the black hole M_0 ,

which are renormalized as $\tilde{\ell} = \ell/\sqrt{\alpha'}$ and $\tilde{M}_0 = M_0/\alpha'^{\frac{D-3}{2}}$, respectively. In what follows, we omit tilde on the variables for simplicity. We have also defined

$$(D-m)_n \equiv (D-m)(D-m-1)(D-m-2)\cdots(D-n),$$

$$h \equiv 1 + 2(D-3)e^{-\gamma\phi} \left[(D-4)\frac{k-B}{r^2} + \gamma\phi' \frac{3B-k}{r} \right], \quad (2.6)$$

$$\tilde{h} \equiv 1 + 2(D-3)e^{-\gamma\phi} \left[(D-4)\frac{k-B}{r^2} + \gamma\phi' \frac{2B}{r} \right], \quad (2.7)$$

$$U(r) \equiv (2\tilde{h})^{-1} \left[(D-3)_4 \frac{k-B}{r^2 B} - 2 \frac{D-3}{r} \left(\frac{B'}{B} - \delta' \right) - \frac{1}{2} \phi'^2 \right. \\ \left. + (D-3)e^{-\gamma\phi} \left\{ (D-4)_6 \frac{(k-B)^2}{r^4 B} - 4(D-4)_5 \frac{k-B}{r^3} \left(\frac{B'}{B} - \delta' - \gamma\phi' \right) \right. \right. \\ \left. - 4(D-4)\gamma \frac{k-B}{r^2} \left(\gamma\phi'^2 + \frac{D-2}{r} \phi' - \Phi \right) + 8 \frac{\gamma\phi'}{r} \left[\left(\frac{B'}{2} - \delta' B \right) \left(\gamma\phi' - \delta' + \frac{2}{r} \right) \right. \right. \\ \left. \left. - \frac{D-4}{2r} B' \right] + 4(D-4) \left(\frac{B'}{2B} - \delta' \right) \frac{B'}{r^2} - \frac{4\gamma}{r} \Phi (B' - 2\delta' B) \right\} - \frac{1}{B} \Lambda e^{\lambda\phi} \right], \quad (2.8)$$

$$\Phi \equiv \phi'' + \left(\frac{B'}{B} - \delta' + \frac{D-2}{r} \right) \phi'. \quad (2.9)$$

This is the system we study in this paper.

§3. Internal Spacetime and Global Structure in GR and Non-dilatonic case

3.1. General Relativity

In the limit of $\alpha \rightarrow 0$, Gauss-Bonnet gravity reduces to GR. By the no-scalar hair theorem in GR, the dilaton field becomes trivial. In this limit,

$$B = k - \frac{2M}{r^{D-3}} + \frac{r^2}{\ell^2}, \quad \delta \equiv 0, \quad (3.1)$$

is obtained. Here $\Lambda = -(D-1)_2/\ell^2$, and M is an integration constant related to the mass of the black hole. In GR, the $(D-2)$ -dimensional constant curvature space can be replaced by any $(D-2)$ -dimensional Einstein space.

The global structure of the spacetime is characterized by the properties of the singularities, horizons, and infinity. Details of the global structure of the solution in GR are discussed in Ref. 13). There are six different types of global structure of the black hole solution. They are Schwarzschild, Schwarzschild-dS, Schwarzschild-AdS, Reissner-Nordström-AdS, and extreme Reissner-Nordström-AdS types. The remaining one is similar to the Schwarzschild-AdS type but the central singularity is replaced by the regular center. In this paper we call it “regular AdS black hole” type.

Here we summarize the behavior of the curvature around the singularity for comparison. There is a curvature singularity at the center ($r = 0$) except for the case with $M = 0$. Around the center, the Kretschmann invariant behaves as follows:

$$\begin{aligned}\mathcal{I} &= R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} \\ &= [B'' - 3B'\delta' + 2B(\delta'^2 - \delta'')]^2 + \frac{2(D-2)}{r^2}(B'^2 - 2BB'\delta' + 2B^2\delta'^2) \\ &\quad + \frac{2(D-2)_3}{r^4}(k-B)^2\end{aligned}\tag{3.2}$$

$$\sim O\left(\frac{M^2}{r^{2D-2}}\right).\tag{3.3}$$

When $M = 0$, the black hole solution exists only for $k = -1$. The Kretschmann invariant is finite at the center $r = 0$ and the spacetime could be regular there.

3.2. Non-dilatonic Case

In the non-dilatonic case ($\gamma = 0$ and $\phi \equiv 0$), the GB term becomes topological invariant and does not contribute to the field equation for $D = 4$. Hence we consider the $D \geq 5$ cases. The gravitational equations give the solution

$$B(r) = k + \frac{1}{2(D-3)_4} \left(1 \mp \sqrt{1 - \frac{4(D-3)_4}{\ell^2} + \frac{8(D-3)_4\bar{M}}{r^{D-1}}} \right) r^2, \quad \delta \equiv 0,\tag{3.4}$$

where $\Lambda = -(D-1)_2/\ell^2$, and \bar{M} is an integration constant related to the mass of the black hole. This is the extended Boulware-Deser solution including the cosmological constant and the topological black hole case.¹²⁾ There are two families of solutions depending on the sign in front of the square root in Eq. (3.4). We call the solution with minus (plus) sign the GR (GB) branch solution because the solution with minus sign reproduces the solution in GR in the $\alpha \rightarrow 0$ limit while there is no such limit for the solution with plus sign. Details of the global structure of this solution are summarized in Ref. 13). Just as in the GR case, there are six different types of global structure of the black hole solutions.

Besides the central singularity at $r = 0$, there can be another type of singularity at $r = r_b > 0$ called the branch singularity. The value r_b is obtained by the condition that the square root term in Eq. (3.4) vanishes, i.e., two branches of solutions coincide. In order for the solution to be well defined, the following condition should be satisfied:

$$\frac{4(D-3)_4}{\ell^2} < 1.\tag{3.5}$$

Under this condition, there is no branch singularity when $\bar{M} \geq 0$, while there exists the branch singularity for $\bar{M} < 0$. We find that r_b is given by

$$r_b^{D-1} = 8(D-3)_4 \left[1 - \frac{4(D-3)_4}{\ell^2} \right]^{-1} |\bar{M}|.\tag{3.6}$$

On the other hand, the positive-mass solutions have only a central singularity. The metric function behaves as

$$B \approx \mp \sqrt{\frac{2\bar{M}}{(D-3)_4 r^{D-5}}} + k + \frac{r^2}{2(D-3)_4} \mp \frac{1}{8} \sqrt{\frac{1}{2[(D-3)_4]^3 \bar{M}}} \left(1 - \frac{4(D-3)_4}{\ell^2}\right) r^{\frac{D+3}{2}}, \quad (3.7)$$

around the center. The Kretschmann invariant behaves as

$$\mathcal{I} \sim O\left(\frac{\bar{M}}{r^{D-1}}\right). \quad (3.8)$$

Although B looks finite for $D = 5$ and there appears to be no singularity at the center, the Kretschmann invariant diverges as

$$\mathcal{I} \sim \frac{6\bar{M}}{r^4} + O(r^{-2}), \quad (3.9)$$

which is the same order as Eq. (3.8), so that the center is singular also in this case.

When the branch singularity exists, the metric function B behaves around it as

$$B(r) \approx \left(k + \frac{r_b^2}{2(D-3)_4}\right) \mp \frac{r_b^2}{2(D-3)_4} \sqrt{\frac{D-1}{r_b} \left(1 - \frac{4(D-3)_4}{\ell^2}\right)} (r - r_b)^{1/2}, \quad (3.10)$$

and the Kretschmann invariant behaves as

$$\mathcal{I} \sim O\left[\frac{1}{(r - r_b)^3}\right]. \quad (3.11)$$

It is interesting to note that the divergent rate does not depend on the dimensions, in contrast to the central singularity.

Note that the divergent behavior of the central singularity in Gauss-Bonnet gravity is milder than that in GR [compare Eqs. (3.3) and (3.8)]. We also point out that the divergent behavior of the branch singularity is milder than that of the central singularity [compare Eqs. (3.11) and (3.8)]. The global structures of the non-dilatonic black holes are classified into six different types which are same as the GR case.

§4. Internal Spacetime and Global Structure in Dilatonic GB Theory

4.1. $\Lambda = 0$ and $k = 1$

Let us proceed to the dilatonic black holes in GB gravity. First one is the solutions with $k = 1$ and no cosmological constant.¹⁵⁾ All of these solutions are asymptotically locally flat, and the mass of the black hole M_0 is defined by

$$B \rightarrow k - \frac{2M_0}{r^{D-3}} \quad (\text{as } r \rightarrow \infty). \quad (4.1)$$

There is no horizon outside of the black hole event horizon. Hence the domain of outer communication of the solution has the same structure as that of Schwarzschild black hole. We will discuss the internal structure of the solution individually for each dimension.

4.1.1. $D = 4$

The internal structure of the dilatonic solution with $\gamma = 1$ in $D = 4$ was investigated in Ref. 6). Here we examine the $\gamma = 1/2$ case.

Integrating the field equations inward from the black hole event horizon with the boundary values which are obtained from the exterior solutions, we find that for any size of black hole, the singularity exists at the nonzero finite radius r_s . The inside of this radius is disconnected from our world. Around this singularity, the field functions behaves as

$$B - B(r_s) \sim (r - r_s)^{1/2}, \quad \delta - \delta(r_s) \sim (r - r_s)^{1/2}, \quad \phi - \phi(r_s) \sim (r - r_s) + (r - r_s)^{3/2}. \quad (4.2)$$

It should be noted that the metric functions and the dilaton field are finite even at $r = r_s$. These behaviors of the field functions around r_s are universal for all the solution in all dimension as we will see later. We call this singularity the fat singularity. The Kretschmann invariant diverges as

$$\mathcal{I} \sim O\left[\frac{1}{(r - r_s)^4}\right]. \quad (4.3)$$

The leading behavior is governed by k in the third term in Eq. (3.2). The divergence is slightly stronger than in the non-dilatonic case. The mechanism of the appearance of the singularity at finite radius will be discussed in the next $D = 5$ case.

We show the relations of r_s and r_H with the black hole mass in Fig. 1. There is a lower limit on the size of the black hole in $D = 4$. The second derivative of the dilaton field diverges at the horizon for the minimum solution. Our analysis shows that the horizon radius and radius of the fat singularity coincide at this limiting size. This means that the existence of the lower limit is due to the appearance of the singularity. Further analysis is necessary to determine whether the singularity of the minimum solution is naked or not. The locations of the r_H and r_s depend on the black hole mass M_0 as $r_H \propto M_0$ and $r_s \propto M_0^{1/3}$, respectively (See Fig. 1 (a)). The physical interpretation of this dependence is under investigation.

The global structure of the spacetime is determined by the properties of the singularity, horizons and infinities. Whether the singularity is spacelike, null, or timelike depends on the dominant term of the metric function B near the singularity. The tortoise coordinate is defined by

$$r_* = \int^r e^{\delta} B^{-1} dr. \quad (4.4)$$

Since the metric function B is negative and the tortoise coordinate is finite at the singularity $r = r_s$, the singularity is spacelike. The spacetime is asymptotically flat. There is no root of $B = 0$ inside of the black hole horizon, so no inner horizon. Hence the global structure of these solutions is the same as that of Schwarzschild black hole.

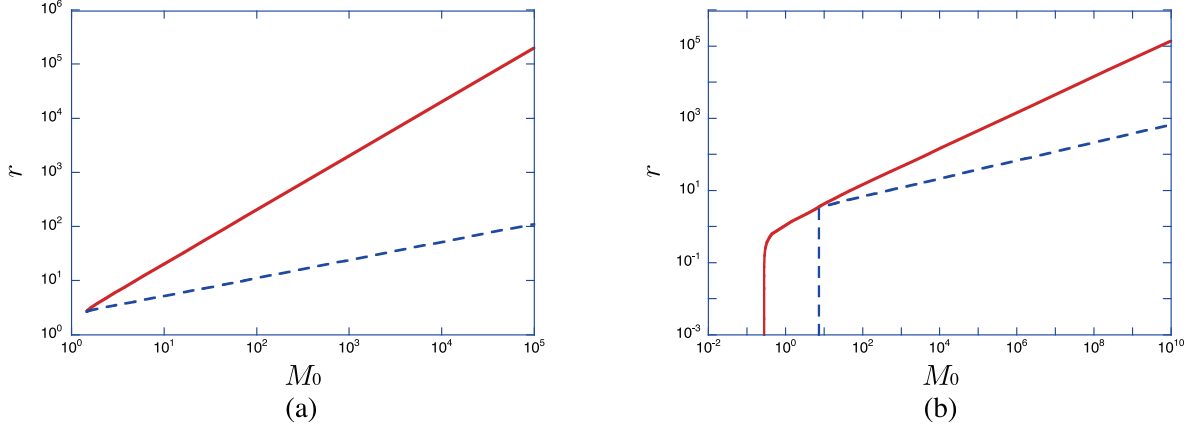


Fig. 1. The relations of r_s (dashed and blue line) and r_H (solid and red line) with M_0 for the black hole solutions in (a) 4 and (b) 5 dimensions with $\gamma = 1/2$, $\Lambda = 0$, $k = 1$.

4.1.2. $D = 5$

For the small black holes with $r_H < 3.46$, the singularity exists at the center and the spacetime is regular for $r > 0$ as in the GR case. Assuming that the power dependence of the functions at the center is given as

$$B \sim r^a, \quad e^{2\delta} \sim r^b, \quad e^{\gamma\phi} \sim r^c, \quad (4.5)$$

and those terms including B and $e^{\gamma\phi}$ in the field equations (2.3)–(2.5) give the leading contributions, we find that $a - c = 2$ in arbitrary dimensions. Our numerical analyses in $D = 5$ indeed show that the field functions behave as

$$B \sim r^{-10.9}, \quad e^{2\delta} \sim r^{-12.9}, \quad e^{\gamma\phi} \sim r^{-12.9}, \quad (4.6)$$

which is independent of the horizon radius. This means that the Kretschmann invariant diverges as

$$\mathcal{I} \sim O\left(\frac{1}{r^{25.8}}\right). \quad (4.7)$$

This is violent divergence, and is much stronger than Schwarzschild solution in GR.

Since the metric function B is negative and the tortoise coordinate is finite at the center, the central singularity is spacelike. There is no inner horizon. Hence the global structure of these solutions is again the same as that of Schwarzschild black hole.

The behaviors of the field functions are shown in Fig. 2. The metric function B monotonically increases inside of the horizon. When the horizon radius becomes as large as $r_H = 3.46$, the denominator in the equation of the dilaton field $\phi'' = \dots$ vanishes at $r \approx 2.9$.

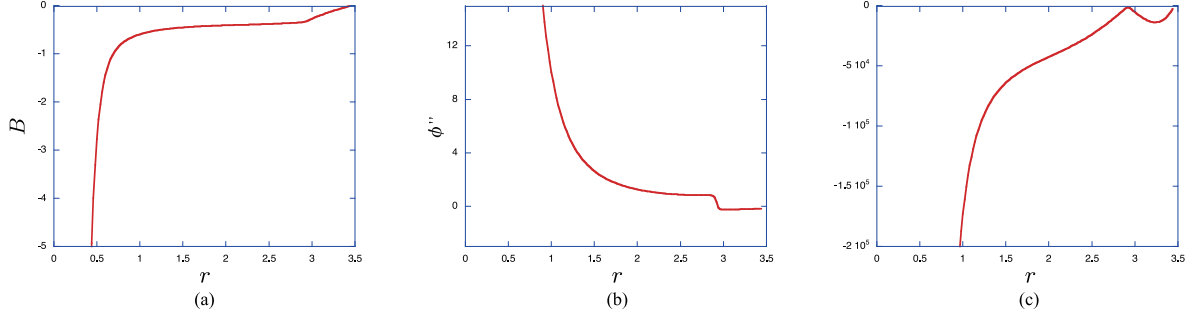


Fig. 2. The configurations of (a) the metric function B , (b) the second derivative of the dilaton field and (c) the denominator of the equation ϕ'' for the solution in 5 dimensions with $\gamma = 1/2$, $\Lambda = 0$, $k = 1$, and $r_H = 3.45$.

In Fig. 2 (c), we show the behavior of the denominator for the value of r_H just before the singularity is formed. At $r_H = 3.46$, the second derivative of the dilaton field diverges and the singularity is formed at this point. This is the fat singularity. This fat singularity locates at nonzero finite radius as the branch singularity in the non-dilatonic case. Its formation is, however, caused by the different mechanism. It is due to the singularity associated with the dilaton. We also see that there is no other solution, hence this is not the branch singularity.

For the large black holes with $r_H > 3.46$, the singularity exists at the nonzero finite radius r_s as in the $D = 4$ case. Around the fat singularity, the field functions behaves as Eq. (4.2). The Kretschmann invariant diverges as

$$\mathcal{I} \sim O\left[\frac{1}{(r - r_s)^4}\right]. \quad (4.8)$$

This is the same divergent rate as the central singularity with positive mass in the non-dilatonic case. The locations of the r_H and r_s depend on the black hole mass M as $r_H \propto M^{1/2}$ and $r_s \propto M^{1/4}$, respectively (See Fig. 1 (b)). Since the tortoise coordinate is finite at $r = r_s$, the singularity is spacelike. Hence the global structure of these solutions is again the same as that of Schwarzschild black hole.

4.1.3. $D = 6 - 10$

In dimensions higher than five, we do not find the fat singularity for any horizon radius. The singularity exists at the center and the spacetime is regular for $r > 0$. There is no inner horizon. Assuming the behavior of the field functions as Eq. (4.5), we plot the exponents a , b , c as functions of r_H in Fig. 3. The divergent rates depend on the horizon radius. In $D = 6$, there is the range $r_H \in (1.613, 5.848)$ where the exponents become constant as

$$B \sim r^{-14.1}, \quad e^{2\delta} \sim r^{-16.1}, \quad e^{\gamma\phi} \sim r^{-16.1}. \quad (4.9)$$

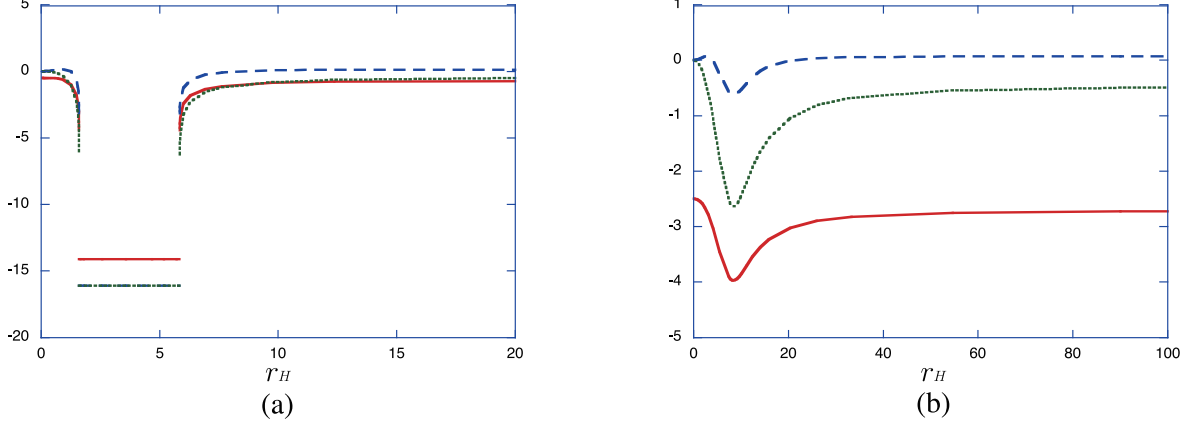


Fig. 3. The divergent rates of the field functions around the central singularity for the black hole solution in (a) 6 and (b) 10 dimensions with $\gamma = 1/2$, $\Lambda = 0$, $k = 1$. We plot the exponents of $B \sim r^a$ (solid and red line), $e^{2\delta} \sim r^b$ (dashed and blue line), and $e^{\gamma\phi} \sim r^c$ (dotted and green line).

We find the similar behavior in $D = 7$:

$$B \sim r^{-17.3}, \quad e^{2\delta} \sim r^{-19.3}, \quad e^{\gamma\phi} \sim r^{-19.3}. \quad (4.10)$$

for the range $r_H \in (3.59, 5.77)$. In this range, the exponents have the relation $a - 2 = c$ as in the $D = 5$ case. By numerical analysis, we find that

$$a = -3.1807D + 4.9683, \quad (4.11)$$

for these ranges. For larger dimensions $D \geq 8$, however, there is no such range with constant exponents as shown in Fig. 3 (b).

Except for the ranges discussed above, the divergent rates depend on the horizon radius. For the large black hole, the exponents approach $a = -0.67$, $b = 0.15$, $c = -0.41$ ($D = 6$), $a = -2.72$, $b = 0.067$, $c = -0.46$ ($D = 10$). For the function B , the mildest divergence appears in the zero horizon limit, where $a = -(D - 5)/2$, $b = 0$, $c = 0$. The exponents changes discontinuously at $r = 1.613$ and $r = 5.848$.

The Kretschmann invariant diverges as

$$\mathcal{I} \sim O(r^{2a-4}). \quad (4.12)$$

Hence the Kretschmann invariant diverges most violently with the rate $r^{-32.2}$ in the range $r_H \in (2.686, 6.884)$ in $D = 6$, and $r^{-11.9}$ for $r_H = 8.26$ in $D = 10$. The mildest divergence is obtained in the zero horizon limit in both dimensions as

$$\mathcal{I} \sim O\left(\frac{1}{r^{D-1}}\right), \quad (4.13)$$

which is the same order as the non-dilatonic one.

Since the tortoise coordinate is finite at $r = 0$, the singularity is spacelike. There is no event horizon except for the black hole horizon. The spacetime is asymptotically flat. Hence the global structure of these solutions is again the Schwarzschild black hole type.

In this paper we consider the model with $\gamma = 1/2$. However, depending on how compactification is made and/or other circumstances, γ can take different values such as $\gamma = \sqrt{2/(D-2)}$. We have confirmed that whether and when the fat singularity appears depends on γ . For example, the fat singularity appears at $r = 7.95307$ for $D = 6$, $\gamma = 1$, and $r_H = 8.49422$. This suggests that the fat singularity is more likely to appear for the larger dilaton coupling.

4.2. $\Lambda < 0$ and $k = 0$

For the solutions with negative cosmological constant,^{16),17)} we impose the “AdS asymptotic behavior” at infinity. It is

$$B \sim b_2 r^2 + k - \frac{2M}{r^\mu}, \quad \delta(r) \sim \delta_0 + \frac{\delta_1}{r^\sigma}, \quad \phi \sim \phi_0 + \frac{\phi_1}{r^\nu}, \quad (4.14)$$

with finite constants $b_2 > 0$, M , δ_0 , δ_1 , ϕ_0 , ϕ_1 and positive constant μ , σ , ν . The coefficient of the first term b_2 is related to the AdS radius as $b_2 = \ell^{-2}$. By analyzing the asymptotic expansion,^{16),17)} we find

$$b_2^2 = \frac{\lambda|\Lambda|}{(D)_3\gamma} \left[\frac{D(D-3)}{(D-1)_2} \frac{\gamma|\Lambda|}{\lambda} \left(1 + \frac{(D-4)\lambda}{D\gamma} \right)^2 \right]^{\frac{\gamma+\lambda}{\gamma-\lambda}}. \quad (4.15)$$

Eq. (4.14) is not, however, sufficient for the spacetime to be the exactly AdS asymptotically. Strictly speaking, the asymptotically AdS spacetime is left invariant under $SO(D-1, 2)$.³⁰⁾ Whether the solution satisfies the AdS-invariant boundary condition or not depends on the value of the power indices μ , σ , and ν . The mass of the solution M_0 is defined by^{*)}

$$-g^{tt} \rightarrow b_2 r^2 + k - \frac{2M_0}{r^{D-3}} \quad (\text{as } r \rightarrow \infty). \quad (4.16)$$

To study black hole solutions for various cosmological constants, it is convenient to note that the field equations have a shift symmetry:^{16),17)}

$$\phi \rightarrow \phi - \phi_*, \quad \Lambda \rightarrow e^{(\lambda-\gamma)\phi_*} \Lambda, \quad B \rightarrow e^{-\gamma\phi_*} B, \quad (4.17)$$

where ϕ_* is an arbitrary constant. This changes the magnitude of the cosmological constant. Hence this may be used to generate solutions for different cosmological constants but with

^{*)} See Ref. 16) for the details.

the same horizon radius, given a solution for some cosmological constant and r_H . Besides the above symmetry, the field equations for $k = 0$ are invariant under the following scaling transformation:

$$B \rightarrow a^2 B, \quad r \rightarrow ar, \quad (4.18)$$

with an arbitrary constant a . If a black hole solution with the horizon radius r_H is obtained, we can generate solutions with different horizon radii but the same Λ by this scaling transformation. Combining these two symmetries, we can find relation between the mass and the horizon radius:

$$M_0 \propto |\Lambda|^{\gamma/(\gamma-\lambda)} r_H^{D-1}. \quad (4.19)$$

These scaling symmetries also means

$$M_0 \propto |\Lambda|^{\gamma/(\gamma-\lambda)} r_s^{D-1}, \quad (4.20)$$

if there is a fat singularity at r_s . As in Ref. 16) we fix the values of the parameters as $\gamma = 1/2$, $\lambda = 1/3$ in the following numerical analysis in this subsection.

4.2.1. $D = 4$ and $D = 5$

The exterior solution is obtained by integrating the field equations from the event horizon with the suitable boundary condition to infinity. Then we find the M_0 - r_H relation as

$$M_0 = 0.0830 |\Lambda|^3 r_H^3 \quad (D = 4), \quad (4.21)$$

$$M_0 = 0.140 |\Lambda|^3 r_H^4 \quad (D = 5). \quad (4.22)$$

On the other hand, by integrating inward from the event horizon, we find the fat singularity at finite radius r_s for any horizon radius. Locations of the singularity are found to be

$$r_s = 0.883 r_H \quad (D = 4), \quad (4.23)$$

$$r_s = 0.827 r_H \quad (D = 5). \quad (4.24)$$

The behaviors of the field functions around the fat singularity are the same as the case with $\Lambda = 0$ and $k = 1$ given in Eq. (4.2). The Kretschmann invariant diverges as

$$\mathcal{I} \sim O\left[\frac{1}{(r - r_s)^3}\right]. \quad (4.25)$$

This divergence is milder than the non-dilatonic case. Since the singularity locates at the finite tortoise coordinate, it is spacelike. Then, the global structure is Schwarzschild-AdS type.

4.2.2. $D = 6 - 10$

The M_0 - r_H relation is given by

$$M_0 = 0.159 |\Lambda|^3 r_H^5 \quad (D = 6), \quad (4.26)$$

$$M_0 = 0.0794 |\Lambda|^3 r_H^9 \quad (D = 10). \quad (4.27)$$

Integrating inward, we find that the singularity exists at the center, and the spacetime is regular for $r > 0$. There is no inner horizon. The field functions behave as

$$B \sim r^{-14.1}, \quad e^{2\delta} \sim r^{-16.1}, \quad e^{\gamma\phi} \sim r^{-16.1} \quad (D = 6), \quad (4.28)$$

$$B \sim r^{-26.8}, \quad e^{2\delta} \sim r^{-28.8}, \quad e^{\gamma\phi} \sim r^{-28.8} \quad (D = 10). \quad (4.29)$$

These violent divergences are similar to the one expressed by Eqs. (4.9) and (4.10) in the $\Lambda = 0$ and $k = 1$ case. The exponent of B is obtained and it is given by Eq. (4.11). The Kretschmann invariant diverges as

$$\mathcal{I} \sim O\left(\frac{1}{r^{32.2}}\right) \quad (D = 6), \quad (4.30)$$

$$\mathcal{I} \sim O\left(\frac{1}{r^{57.6}}\right) \quad (D = 10). \quad (4.31)$$

The singular behavior at the center is quite strong. The singularity is spacelike, and the global structure is Schwarzschild-AdS type.

4.3. $\Lambda < 0$ and $k = 1$

As we will see below, the internal structures of the black hole solution with $\Lambda < 0$ and $k = 1$ ¹⁷⁾ are qualitatively the same as those in the $\Lambda = 0$ and $k = 1$ case. This implies that the cosmological constant does not affect the internal structure so much as we expected. All the black hole solutions are locally AdS at infinity. The inverse square of the AdS radius b_2 is given by Eq. (4.15).

4.3.1. $D = 4$

In $D = 4$, there are minima for the horizon radius ($r_H = 3.245$) and the mass of the solution, below which the black holes cease to exist. For the large black hole, the spacetime approaches the GR one. Integrating the field equations inward, we find the second derivative of the dilaton field diverges at finite radius r_s for any black hole solution. The formation mechanism of this fat singularity is the same as the zero cosmological constant case. The divergent rate of the Kretschmann invariant is given by

$$\mathcal{I} \sim O\left[\frac{1}{(r - r_s)^4}\right]. \quad (4.32)$$

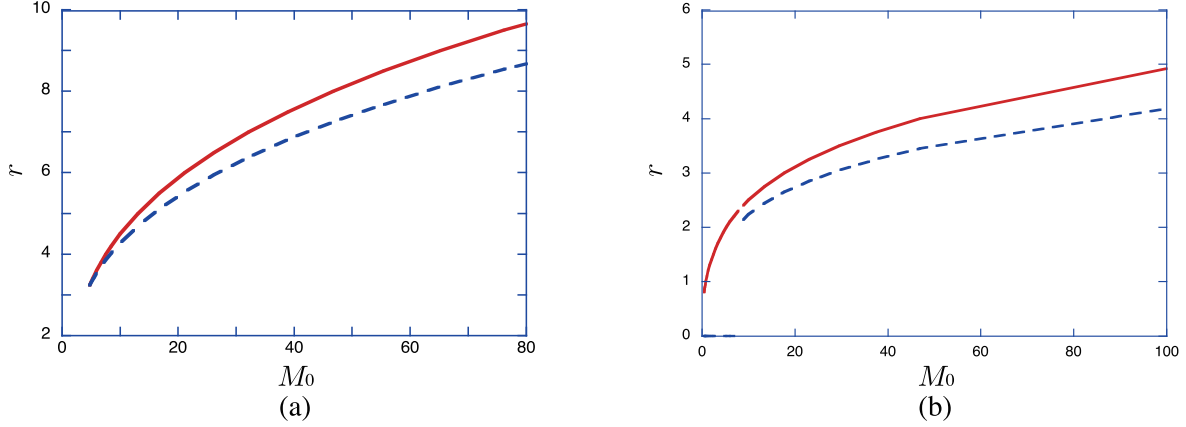


Fig. 4. The r_s - M_0 (dashed and blue line) and r_H - M_0 (solid and red line) diagrams for the black hole solutions in (a) 4 and (b) 5 dimensions with $\gamma = 1/2$, $\Lambda = -1$, $k = 1$.

For the minimum mass solution, we find that r_s and r_H coincide with each other, and the fat singularity appears at the location of the event horizon. We show the M_0 - r_H and M_0 - r_s diagrams in Fig. 4 (a). The global structure is the Schwarzschild-AdS type.

4.3.2. $D = 5$

In $D = 5$, there are also minima for the horizon radius ($r_H = 0.805$) and the mass of the solution. There is the fat singularity at $r = r_s$ for the large black holes $r_H > 2.35$ as depicted in Fig. 4 (b). At the singularity, the Kretschmann invariant diverges as Eq. (4.32). On the other hand, for the small black holes $r_H < 2.35$, the singularity locates at the center. The field functions behave as

$$B \sim r^{-10.9}, \quad e^{2\delta} \sim r^{-12.9}, \quad e^{\gamma\phi} \sim r^{-12.9}, \quad (4.33)$$

and the Kretschmann invariant becomes

$$\mathcal{I} \sim O\left(\frac{1}{r^{25.8}}\right). \quad (4.34)$$

These divergent rates are again same as the zero cosmological constant case [see Eqs. (4.6) and (4.7)]. For the minimum mass solution, the radii of the event horizon and singularity are different. This means that the disappearance of the black hole solution at $r_H = 0.805$ is not due to the coincidence of the internal singularity with the event horizon. Actually, if we integrate outward from $r = 0.805$ with the boundary condition of the event horizon, the spacetime just outside of $r = 0.805$ is regular. However, continuing the integration, we find that the second derivative of the dilaton field diverges at $r = 1.119$ and the spacetime becomes singular. Hence, the M_0 - r_H and M_0 - r_s curves are disconnected.

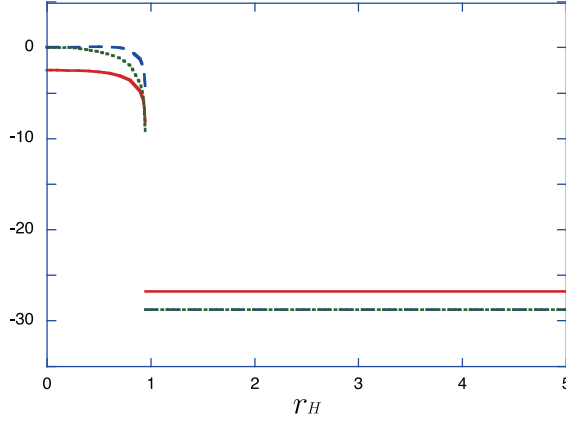


Fig. 5. The divergent rates of the field functions around the central singularity for the black hole solution in 10-dimensions with $\gamma = 1/2$, $\Lambda = -1$, $k = 1$. We plot the exponents of $B \sim r^a$ (solid line), $e^{2\delta} \sim r^b$ (dashed line), and $e^{\gamma\phi} \sim r^c$ (dotted line).

Since the tortoise coordinate is finite at $r = 0$, the singularity is spacelike. There is no event horizon except for the black hole horizon. The spacetime is asymptotically AdS. Hence the global structure is again the Schwarzschild-AdS black hole type.

4.3.3. $D = 6 - 10$

As in the $D = 5$ case, there is the minimum horizon radius $r_H = 0.452$ for the black hole solution in $D = 6$, while we can take $r_H \rightarrow 0$ limit in $D = 10$. The outer spacetime is almost the same as that in the $k = 0$ case in the large horizon limit. The singularity exists at the center and the spacetime is regular for $r > 0$ both in $D = 6$ and 10.

In $D = 6$, the field functions behave as Eq. (4.28) independently of the horizon radius, and the Kretschmann invariant diverges as

$$\mathcal{I} \sim O\left(\frac{1}{r^{32.2}}\right) \quad (D = 6). \quad (4.35)$$

In $D = 10$, the divergent rates of the field functions depend on the horizon radius as depicted in Fig. 5. For $r_H > 0.949$, the exponents become constant and

$$B \sim r^{-26.8}, \quad e^{2\delta} \sim r^{-28.8}, \quad e^{\gamma\phi} \sim r^{-28.8} \quad (D = 10), \quad (4.36)$$

as in the $k = 0$ case. For $r_H < 0.949$, the exponents depend on the horizon radius. The mildest divergence of the function B appears in the zero horizon limit, where $a = -(D - 5)/2$, $b = 0$, $c = 0$. The Kretschmann invariant diverges as

$$\mathcal{I} \sim O(r^{2a-4}) \quad (D = 10), \quad (4.37)$$

and the mildest divergence in $D = 10$ is obtained in the zero horizon limit as

$$\mathcal{I} \sim O\left(\frac{1}{r^{D-1}}\right), \quad (4.38)$$

which is the same order as the non-dilatonic one.

Since the tortoise coordinate is finite at $r = 0$, the singularity is spacelike. There is no event horizon except for the black hole horizon. The spacetime is asymptotically AdS. Hence the global structure of these solutions is again the Schwarzschild-AdS black hole type.

4.4. $A < 0$ and $k = -1$

In the $k = -1$ case, the qualitative properties are the same in all dimensions. Hence we can discuss $D = 4 - 6$ and 10 cases simultaneously.

Here the basic equations have the exact solution¹⁷⁾

$$\phi \equiv \phi_0 = \text{constant}, \quad \delta \equiv 0, \quad B = b_2 r^2 - 1. \quad (4.39)$$

This solution has a black hole event horizon at $r = 1/\sqrt{b_2}$ which is the AdS radius. The mass of the black hole is zero and the Kretschmann invariant is finite everywhere. The global structure of this zero mass black hole is the “regular AdS black hole” type.

Except for the zero mass black hole solution, the singularity appears at the nonzero finite radius r_s where the second derivative of the dilaton field ϕ'' diverges. Fig. 6 shows the radii r_s and r_H as functions of the black hole mass. The behavior of the field functions is given by Eq. (4.2), and the Kretschmann invariant becomes

$$\mathcal{I} \sim O\left[\frac{1}{(r - r_s)^4}\right]. \quad (4.40)$$

In the non-dilatonic case, the Kretschmann invariant is $\mathcal{I} \sim O(r^{-(D-1)})$ for the central singularity of the positive mass black hole while $\mathcal{I} \sim O[(r - r_b)^{-3}]$ for the branch singularity of the negative mass black hole. Compared with the non-dilatonic case, the diverging behavior becomes mild for the positive mass solutions in $D = 6 - 10$. On the other hand, it is stronger for the positive mass solutions in $D = 4$ and negative mass solutions in all dimensions.

The black hole solution has critical horizon radius $r_H = 1.807, 1.712, 1.777$, and 2.364 , for $D = 4, 5, 6$, and 10 , respectively, below which there is no black hole solution. At the critical horizon radius, the radii of the horizon and the singularity coincide and the horizon becomes singular. It appears in Fig. 6 as if the curves of r_H and r_s is disconnected. However, this is just due to the difficulty of the numerical analysis around the critical horizon radius, and they should be connected.

Since the tortoise coordinate is finite at $r = 0$, the singularity is spacelike. There is no event horizon except for the black hole horizon. The spacetime is asymptotically AdS. Hence the global structure of these solutions is again the Schwarzschild-AdS black hole type.

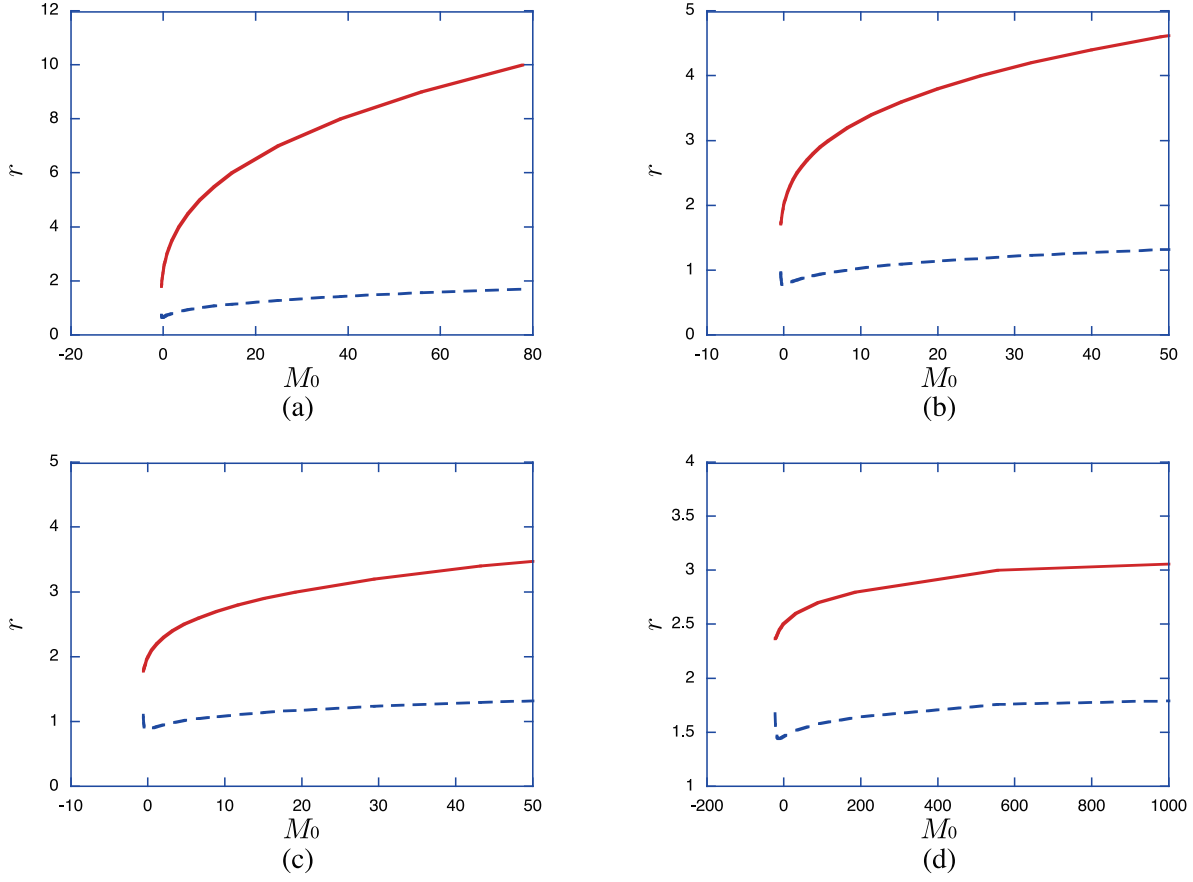


Fig. 6. The r_s - M_0 (dashed and blue line) and r_H - M_0 (solid and red line) diagrams for the black hole solutions in (a) 4, (b) 5, (c) 6, and (d) 10 dimensions with $\gamma = 1/2$, $\Lambda = -1$, $k = -1$.

§5. Conclusions and Discussions

We have investigated how the dilaton field affects the global structure of the black hole solution in Einstein-Gauss-Bonnet gravity. In this paper the system includes the zero and negative cosmological constant and the spacetime is assumed to have spherical, planar, and hyperbolic symmetry. The global structure of the solution is changed by the effect of the dilaton field drastically. In the non-dilatonic system, the solutions have six different types of global structure while there are just three types in the dilatonic system. They are the Schwarzschild type for the zero cosmological constant, the Schwarzschild-AdS type in the negative cosmological constant case, and the “regular AdS black hole” type for the zero mass black hole.

One of the important issues is whether the dilaton field makes the strength of the singularity mild or not. We have investigated the Kretschmann invariant around the singularity in detail. We have found that the singularity is classified into three types. One is the

central singularity around which the divergent rate of the Kretschmann invariant does not depend on the horizon radius as in the non-dilatonic case. The second one is the central singularity around which the divergent rate depends on the horizon radius. The third one is the fat singularity, which locates at the non-zero finite radius. All types of singularities are spacelike. The divergent rates of the Kretschmann invariant are summarized in Table I and II. The first type of the central singularity has violently divergent behavior with the rate $\mathcal{I} \sim r^{-6.36D+5.94}$. On the other hand, the fat singularity has rather mild behavior. We find that there are cases where the singularity becomes mild by the presence of the dilaton field in the system. There is also some dependence on the dilaton coupling whether this mild fat singularity appears or not. We find an indication that it is more likely to appear for the larger dilaton coupling.

As for the event horizon, the black holes solution has just one horizon in the dilatonic system generically. It is due to the structure of the equation of the dilaton field. The equation of the dilaton field (2.5) is rewritten as

$$B\phi'' = -\frac{1}{r^{D-2}e^{-\delta}}(r^{D-2}e^{-\delta}B)'\phi' + (\gamma e^{-\gamma\phi}R_{GB}^2 + \lambda Ae^{\lambda\phi}). \quad (5.1)$$

At the horizon $B = 0$, this equation, when solved for ϕ'' , is singular and the right hand side should vanish. By this condition, ϕ and ϕ' cannot be free but should be related with each other at the horizon. As a result, there is one free parameter $\phi(r_H)$ at the black hole event horizon. This parameter is determined by the asymptotic condition at $r \rightarrow \infty$. However, if there were other event horizons such as the inner horizon and the cosmological horizon, we should have additional conditions between ϕ and ϕ' at these horizons. It is difficult, if not impossible, to satisfy the conditions at some horizons and at infinity simultaneously in a generic occasions. For this reason, our solutions have only single horizon i.e., black hole event horizon generically. This situation seems to remain the same even if we add the charge to the black hole.

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Table I. Summary of the divergent rate of the Kretschmann invariant \mathcal{I} around the singularity in the $\Lambda = 0$ case. We also show those in GR and non-dilatonic cases for comparison.

k	D	\mathcal{I}	\mathcal{I} (GR)	\mathcal{I} (non-dilatonic)
1	4	$(r - r_s)^{-4}$	$r^{-(2D-2)}$	$r^{-(D-1)}$
	5	$r^{-25.8}$ ($M_0 < 7.46$)		
		$(r - r_s)^{-4}$ ($M_0 > 7.46$)		
	6	$r^{-32.2}$ ($5.18 < M_0 < 120.9$) $r^{-(D-1)}$ (otherwise)		
	10	$r^{-(D-1)} < \mathcal{I} < r^{-11.9}$		

Table II. Summary of the divergent rate of the Kretschmann invariant \mathcal{I} around the singularity in the $\Lambda = -1$ case. We also show those in GR and non-dilatonic cases for comparison.

k	D	\mathcal{I}	\mathcal{I} (GR)	\mathcal{I} (non-dilatonic)
1	4	$(r - r_s)^{-4}$	$r^{-(2D-2)}$	r^{-6}
	5	$r^{-25.8}$ ($M_0 < 8.39$) $(r - r_s)^{-4}$ ($M_0 > 8.39$)		$r^{-(D-1)}$
		6		
	10	$r^{-(D-1)} < \mathcal{I} < r^{-21.0}$ ($M_0 < 1.93$) $r^{-57.6}$ ($M_0 > 1.93$)		
	0	4		$(r - r_s)^{-3}$
5		$r^{-(D-1)}$		
6			$r^{-32.2}$	
10			$r^{-57.6}$	
	4	$(r - r_s)^{-4}$ ($M_0 \neq 0$) r^0 ($M_0 = 0$)	$r^{-(2D-2)}$ ($M_0 \neq 0$) r^0 ($M_0 = 0$)	r^{-6} ($M_0 \neq 0$), r^0 ($M_0 = 0$)
	5		$(r - r_b)^{-3}$ ($M_0 < 0$)	
	6		r^0 ($M_0 = 0$)	
	10		$r^{-(D-1)}$ ($M_0 > 0$)	

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